PS/EC 172, HOMEWORK 3
DUE THURSDAY, FEBRUARY 1ST

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *Chicken and correlated equilibria.* Consider the game of Chicken, as parametrized by a, b, c, with $a, b, c > 0$ and $b > a$:

	Y	D
Y	a, a	$0, b$
D	$b, 0$	$-c, -c$

(a) *20 points.* Explain why every finite game has a correlated equilibrium.

(b) *20 points.* Calculate the set of (mixed and pure) Nash equilibria for every possible set of parameters. For which values of a, b, c does there exist a mixed equilibrium with total expected utility larger than that of any pure equilibrium?

(c) *20 points.* For which values of a, b, c does there exist a symmetric correlated equilibrium with total expected utility larger than that of any pure equilibrium? In this game, a correlated equilibrium is symmetric if the probabilities of (Y, D) and (D, Y) are the same.

(2) *Zero-sum games.*

(a) *10 points.* For any two given numbers $0 < p \leq q < 1$, find a two player, two strategy, zero-sum game in which one player mixes with probabilities p and $1 - p$ and the other with probabilities q and $1 - q$ in the unique mixed Nash equilibrium.

(3) *Equilibria.*

(a) *20 points.* Let G be a finite extensive form game with perfect information in which each player plays at one history only. That is, for each i there is a single history $h \in H$ such that $P(h) = i$.
Prove or disprove: every pure trembling hand perfect equilibrium of the strategic form of G is a subgame perfect equilibrium of G.
Hint: this is true. To prove this, assume that some pure strategy profile s is not subgame perfect. Find a player i who has a profitable deviation t_i. Show that t_i is also a profitable deviation to any completely mixed σ that is close enough to s, and thus s cannot be trembling hand perfect.

(b) *Bonus question (1 point).* Prove or disprove: every pure subgame perfect equilibrium of a finite extensive form game with perfect information is a trembling hand perfect equilibrium of the strategic form of the game.

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(4) *Very weakly dominant strategies.* Say that a strategy s_i in a normal form game is *very weakly dominant* if for any t_i and any s_{-i} it holds that $u_i(s_{-i}, s_i) \geq_i u_i(s_{-i}, t_i)$.

(a) *2 points.* Find a two player game with a very weakly dominant strategy and an equilibrium in which this strategy is not played.

(b) *8 points.* Explain why every finite two player game that has a very weakly dominant strategy has a pure equilibrium in which this strategy is played.

(5) *Bonus question.* A prisoner escapes to the number line. He chooses some $n \in \mathbb{Z}$ to hide on the zeroth day. He also chooses some $k \in \mathbb{Z}$, and every day hides at a number that is k higher than in the previous day. Hence on day $t \in \{0, 1, 2, \ldots\}$ he hides at $n + k \cdot t$.

Every day the detective can check one number and see if the prisoner is there. If he is there, she wins. Otherwise she can check again the next day.

Formally, the game played between the prisoner and the detective is the following. The prisoner's strategy space is $\{(n, k) : n, k \in \mathbb{Z}\}$, and the detective's strategy space is the set of sequences $(a_0, a_1, a_2, \ldots)$ in $\mathbb{Z}$. The detective wins if $a_t = n + k \cdot t$ for some t. The prisoner wins otherwise.

(a) *1 point.* Prove that the detective has a winning strategy.